The Space-Time Deformation Ensemble Generation Technique

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The space-time deformation ensemble

Forecast errors often well described in terms of shifting and timing errors, i.e.

\[ u'(x, y, z, t) \sim u'(x + \xi, y + \psi, z, t + \tau). \]

\(\xi, \psi\) and \(\tau\) define the space-time shift required to render the forecast "close" to the truth. Can't predict \(\xi, \psi\) and \(\tau\) associated with particular error but can estimate their distribution.

Hoffman et al. (1995)

- To ensure the deformation in space-time is smooth, we must ensure that spatial and temporal shifts are correlated in space and time.
- To ensure that the space-time shifted field is balanced, one would need to use a balancing procedure (e.g. NMI)
Space-time deformation stochastic perturbation

Surface pressure change due to horizontal shift.

normal

shifted
- **Typical problem of “raw” ensembles**

  Observations fall outside the range of ensemble with a margin and frequency that cannot be explained by observational errors.

- **Postprocess ensemble with space-time deformation.**

  Randomly deform high resolution control forecast and then add randomly selected raw ensemble perturbation. Repeat this process many times.

- **Motivation**

  Enables inexpensive very large, very high resolution ensemble forecasts.

  Account for aspects of model error leading to timing and position errors.

  Enables prototype huge ensemble data assimilation systems.

  Feed user application functions easily.
Initialization of a smooth coordinate deformation

Let

\[ \delta x_i = \sum_{j=1}^{N} a_{ij} e_j, \quad (1) \]

where \( e_j \) is the jth eigenvector of a user specified x-shift covariance matrix \( \langle \delta x_i \delta x_i^T \rangle = S \) and \( a_{ij} \sim N(0, \lambda_j); i.e. \) it is normally distributed with variance equal to the jth eigenvalue of \( S \).

Given the SVD, \( S = E \Lambda E^T \), eq. (1) can be rewritten as

\[ \delta x_i = E a_i, \quad \text{where} \quad a_i^T = [a_{i1}, a_{i2}, \ldots, a_{ij}, \ldots, a_{iM}] \]

and hence

\[ \langle \delta x_i \delta x_i^T \rangle = E \langle a_i a_i^T \rangle E^T = E \Lambda E^T = S; \]

consequently, the expected covariance of random shifts is consistent with the specified shift covariance matrix.
Propagation of coordinate deformation through time

Solve discrete version of

\[ \frac{da_{ij}}{dt} = -\frac{a_{ij}}{\tau} + \zeta, \text{ where } \zeta \sim N(0, Q) \]

to get time series of coefficients that are correlated in time. Choose discrete counterpart of \( Q \) so that the time average \( \bar{a}_{ij}^2 = \lambda_j \).

This ensures that (a) the temporal correlation of coefficients is consistent with the decorrelation time scale \( \tau \) and (b) the time averaged coefficient magnitude is consistent with the specified coordinate shift covariance matrix, \( S \).
First 4 eigenvectors of covariance between x-shift at different heights. (Correlation function is Gaussian in log(p)).
Eigenvectors 5-8 of covariance between x-shift at different heights. (Correlation function is Gaussian in log(p)).
Eigenvectors 9-12 of covariance between x-shift at different heights. (Correlation function is Gaussian in log(p)).
Eigenvectors 13-16 of covariance between x-shift at different heights. (Correlation function is Gaussian in log(p)).
Top panel gives icosehedral grid (Thanks to Frank Giraldo, NRL, Monterey). Grid point density affects eigenvector structure. Icosehedral grid gives relatively homogeneous resolution over globe.

Bottom panel panel gives covariance of a vertical eigenvector coefficient with vertical eigenvector coefficient at pole. (Correlation function is Gaussian in log(great circle distance)).
Eigenvector 1 of matrix of horizontal covariances of vertical eigenvector coefficients
Eigenvector 2 of matrix of horizontal covariances of vertical eigenvector coefficients
Random correlated x-shift field at 1100 hPa
Random correlated x-shift field at 1045 hPa
Random correlated x-shift field at 276 hPa
Random correlated x-shift field at 221 hPa
800 hPa x-shift field at t=0 hr
800 hPa x-shift field at t=1 hr
800 hPa x-shift field at t=3 hr
800 hPa x-shift field at t=5 hr
Boundary of mesoscale model
Summary

• The space-time deformation technique enables rapid and inexpensive generation of very large, very high resolution ensemble forecasts that can account for timing and position errors. Potential uses include
  • (a) ensemble forecasting,
  • (b) huge ensemble data assimilation,
  • (c) hi-res BC ensembles for mesoscale LAM ensembles, and
  • (d) hi-res surface forcing ensembles for ocean model ensembles.
Issues

• How to specify covariance functions for space-time shifts?
• Is initialization necessary for balance?
• Can it improve error covariance estimates?