The new Local Ensemble Kalman Filter: an optimal method for data assimilation, ensemble forecasting and adaptive observations

Eugenia Kalnay
Department of Meteorology and Chaos Group
University of Maryland

Chaos group at the University of Maryland:
Profs. Szunyogh, Kostelich, Ott, Hunt, Sauer, Yorke, Kalnay
and now Ricardo Todling…
Former students: Drs. Patil, Corazza, Zimin, Gyarmati, Oczkowski
Current students: Yang, Miyoshi, Klein, Danforth, Li, Liu, Merkova
References and thanks:


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Components of ensemble forecasts

An ensemble forecast starts from initial perturbations to the analysis…
In a good ensemble “truth” looks like a member of the ensemble
The initial perturbations should reflect the analysis “errors of the day”
Ensemble forecasting methods

• Early methods:
  – Monte Carlo Forecasting (Leith, 1974)
  – Lagged Average Forecasting (Hoffman and Kalnay, 1983)

• Operational methods:
  – Singular vectors (ECMWF)
  – Breeding (NCEP, JMA, India, others)
  – Ensemble of data assimilations (Canada)
  – Multisystems or poor person (FSU, UKMO,…)

Even though the initial perturbations should be analysis errors, ensemble systems and data assimilation are done independently!!
Example of a very predictable 6-day forecast, with “errors of the day”

Errors of the day tend to be localized and have simple shapes
(locally low ensemble dimension, Patil et al, 2001)
The errors of the day are instabilities of the background flow. At the same verification time, the forecast uncertainties have the same shape.
Strong instabilities of the background tend to have simple shapes (perturbations lie in a low-dimensional subspace).

2.5 day forecast verifying on 95/10/21.

Note that the bred vectors (difference between the forecasts) lie on a 1-D space (red line).

One observation would be enough to choose the right solution!

This suggests that the assimilation and ensemble problems are related…
Ideal ensemble forecast perturbations

“Perfect” initial perturbations should sample well the analysis errors:

$$\frac{1}{K-1} \sum_{i=1}^{K} \delta x_i \delta x_i^T = A$$

The ideal initial perturbations should have a covariance that represents the analysis error covariance $A$, but the problem has been that we do not know $A$, which changes with the “errors of the day”

With the new approaches to Ensemble Kalman Filtering we are able to determine $A$ and the ideal initial perturbations simultaneously.
One approach to create initial perturbations for ensemble forecasting with errors of the day: breeding

- Breeding is simply running the nonlinear model a second time, from perturbed initial conditions, and rescaling the perturbations.
Evolution of Operational Data Assimilation

- Successive Correction Method: empirical weights
- Optimal Interpolation: Gandin, 1965: statistical weights
- The influence of Phillips (1981, 1982, 1986) was essential in replacing SCM with OI.
- OI assumes a constant background error covariance $B$
- OI replaced by 3-D Var in the 90’s (same theoretical solution, $B$ assumed constant)
- 4D-Var implemented at ECMWF and MeteoFrance in the late 1990’s. Requires TLM and Adjoint models.
- $B$ evolves in 4D-Var but it is not explicitly computed (hence need for reduced rank Kalman Filter).
3D-Var used in operational forecasting centers

\[ J = \min \frac{1}{2} [ (x_b - x_a)^T B^{-1} (x_b - x_a) + (y_o - Hx_a)^T R^{-1} (y_o - Hx_a) ] \]

Distance to forecast \hspace{1cm} Distance to observations

- \( x \) is a model state vector, with 10^6-8 d.o.f. \( x_a \) minimizes \( J \)
- \( y_o \) is the set of observations, with 10^5-9 d.o.f.
- \( R \) is the observational error covariance
- \( B \) the forecast error covariance.
- In 3D-Var \( B \) is assumed to be constant: it does not include “errors of the day”
- The methods that allow \( B \) and \( A \) to evolve are very expensive: 4D-Var and Kalman Filtering.
The 3D-Var analysis is given by

\[ x_a = x_b + K(y_o - Hx_b) \]

where the weight matrix is

\[ K = (B^{-1} + H^T R^{-1} H)^{-1} H^T R^{-1} \]

and the analysis error covariance is given by

\[ A = (I - KH)B \]

• In 3D-Var \( B \) is assumed to be constant: it does not include “errors of the day”
• 4D-Var is very expensive and does not provide the analysis error covariance.
• In Kalman Filtering \( B \) is forecasted. It is like running the model \( N \) times, where \( N \sim 10^{6-8} \), so that it is impractical without simplifications.
The solution: Ensemble Kalman Filtering

1) Perturbed observations and ensembles of data assimilation
   • Evensen, 1994
   • Houtekamer and Mitchell, 1998

2) Square root filter, no need for perturbed observations:
   • Tippett, Anderson, Bishop, Hamill, Whitaker, 2003
   • Anderson, 2001
   • Whitaker and Hamill, 2002
   • Bishop, Etherton and Majumdar, 2001

3) Local Ensemble Kalman Filtering: done in local patches or cubes
Suppose we have a 6hr forecast (background) and new observations. The 3D-Var Analysis doesn’t know about the errors of the day. Observations ~10^{5-7} d.o.f. Background ~10^{6-8} d.o.f.
Example from a QG simulation (Corazza et al, 2003)

Background error and 3D-Var analysis increment, June 15

The 3D-Var does not capture the errors of the day
With Ensemble Kalman Filtering we get perturbations pointing to the directions of the "errors of the day."

Observations $\sim 10^{5-7}$ d.o.f.

Background $\sim 10^{6-8}$ d.o.f.

3D-Var Analysis: doesn’t know about the errors of the day.

Errors of the day: they lie on a low-dim attractor.
Ensemble Kalman Filtering is efficient because matrix operations are performed in the low-dimensional space of the ensemble perturbations.

- Observations ~$10^{5-7}$ d.o.f.
- Background ~$10^{6-8}$ d.o.f.
- 3D-Var Analysis: doesn’t know about the errors of the day
- Errors of the day: they lie on a low-dim attractor
- Ensemble Kalman Filter Analysis: correction computed in the low dim attractor
Background error (color) and LEKF analysis increments (contours), June 15

The LEKF makes better use of the obs. because it includes the errors of the day.
Example from a QG simulation (Corazza et al, 2003)

Background error and 3D-Var analysis increment, June 15

The 3D-Var does not capture the errors of the day
After the EnKF computes the analysis and the analysis error covariance $\mathbf{A}$, the new ensemble initial perturbations $\delta \mathbf{a}_i$ are computed:

$$
\frac{1}{k} \sum_{i=1}^{k+1} \delta \mathbf{a}_i \delta \mathbf{a}_i^T = \mathbf{A}
$$

These perturbations represent the analysis error covariance and are used as **initial perturbations** for the next ensemble forecast.
The process is repeated: an ensemble of forecasts is started from each of the initial perturbed analyses and integrated for 6 hours. The new background is the average of the forecasts, and the new low-dimensional attractor is given by the forecast perturbations.
The “local” ensemble Kalman Filter

• In the Local Ensemble Kalman Filter we compute the generalized “bred vectors” globally but use them locally:
  • 3D cubes around each grid point of ~800km x 800km x few layers.
  • These local cubes provide the local shape of the “errors of the day”.
• At the end of the local analysis we create a new global analysis and initial perturbations from the solutions obtained at each grid point (the square-root problem): $\frac{1}{k} \sum_{i=1}^{k+1} \delta a_i \delta a_i^T = A$
  • This reduces the number of ensemble members needed.
  • It also allows to compute the KF analysis independently at each grid point (“embarrassingly parallel”).
Results with Lorenz 40 variable model

- Used by Whitaker and Hamill (2002) to validate their ensemble square root filter (EnSRF)
- A very large global ensemble Kalman Filter converges to an “optimal” analysis rms error=0.20
- This “optimal” rms error is achieved by the LEKF for a range of small ensemble members
- We performed experiments for different size models: M=40 (original), M=80 and M=120, and compared a global KF with the LEKF
With the global EnKF approach, the number of ensemble members needed for convergence increases with the size of the domain $M$.

With the local approach, the number of ensemble members remains small.
Why is the local analysis more efficient?

Schematic of a system with 3 independent regions of instability, A, B and C. Each region can have either wave #1 or #2 instability.

From a local point of view, BV1 and BV2 are enough to represent all possible states.

From a global point of view, BV2 and BV3 are independent, and there are many possible different states…
The LEKF algorithm:

1. Make a 6hr ensemble forecast with $K+1$ members. At each grid point $i$ consider a local 3D volume of $\sim 800$km by 800km and a few layers.

2. The expected value of the background is $\overline{x}_i^b$, the ensemble average, and the $\delta x_i^b = x_i^b - \overline{x}_i^b$ form the background error covariance $B$. In the subspace of the perturbations, $B$ is diagonal, with rank $\leq K$.

3. Use all the observations in the volume and solve exactly the Kalman Filter equations. This gives the analysis $\overline{x}_i^a$ and the analysis error covariance $A$ at the grid point $i$.

4. Solve the square root equation $\delta x_i^a \delta x_i^{aT} = A$ and obtain the analysis increments at the grid point $i$.

5. Transform back $\delta x_i^a$ to the grid-point coordinates

6. Create the new initial conditions for the ensemble $x_{ki}^a = \overline{x}_{ki}^a + \delta x_{ki}^a$

7. Go to 1
Preliminary LEKF results with NCEP’s global model

- T62, 28 levels (1.5 million d.o.f.)
- The method is model independent: essentially the same code was used for the L40 model as for the NCEP global spectral model
- Simulation with observations at every grid point (1.5 million obs)
- Very parallel! Each grid point analysis done independently
- Very fast! 6 minutes in cluster of PCs with 40 ensemble members
From Szunyogh, Kostelich et al

**Results with NCEP’s global model**  
**(perfect model simulation)**

A) observations at every grid point
- With 40 members and no tuning, the rms error was half of the observations rms error

B) observations were thinned until only 2% of the grid points had observations
LEKF using 40 ensemble members:
Analysis temperature errors

100% coverage

11% coverage (~NH)

2% coverage (~SH)

obs. errors
LEKF using 40 ensemble members:
Analysis zonal wind errors

- 100% coverage
- 11% coverage (~NH)
- 2% coverage (~SH)
- obs. errors
RMS temperature analysis errors

11% coverage
RMS zonal wind analysis errors

11% coverage
Advantages of LEKF

• It knows about the “errors of the day” through B.
• Provides perfect initial perturbations for ensemble forecasting.
• Free 6 hr forecasts in an ensemble system
• Matrix computations are done in a very low-dimensional space: both accurate and efficient.
• Extended to 4DLEKF, for asynchronous observations (Sauer et al, 2004, Hunt et al, 2004)
• Does not require adjoint of the NWP model (or the observation operator)
• Can keep track of whether the number of ensemble members is sufficient (E-dimension)
• Can be used for adaptive observations
In the tropics $B$ and $A$ are large because of fast convective growth. In the extratropics, they are large where errors grow due to baroclinic instabilities.
January 16 2000: On the left, we see areas with large trace of $B$, indicating large forecast uncertainty. The tropics dominate because convective instabilities are fast. The right figure (E-dimension) shows that the tropical errors have large effective ensemble dimension. In mid-latitudes, by contrast, the areas of large $B$ are associated with baroclinic instabilities, and have LOW E-dimension. *These are prime areas for targeting.* Also, knowing the local dimension of the ensemble allows for additional tuning of the system such as an inflation factor $>1$ when E-dim~$K$
Preliminary “climatology” of the areas of large forecast uncertainty: Average trace of B in January
Comparison of trace of B and E-dimension for January

Again, we find that on the monthly average, the tropical (convective) growth is associated with high dimensionality (more random directions in the forecast uncertainty), and in mid-latitudes the growth due to baroclinic instability is associated with relatively low dimensionality.
In summary…

- The new Local Ensemble Kalman Filter is accurate and efficient: very parallel
- It provides both an optimal analysis and ideal initial ensemble perturbations
- It does not require linear tangent or adjoint models
- It does not require the adjoint of the observation operator!
- Can be easily extended to 4D EnKF (Sauer et al, Hunt et al)
- **It makes very easy to perform adaptive observations:** the lidar instrument should simply dwell where the errors are large!

**BUT,**

- An important remaining problem is how to handle model deficiencies
- LEKF may also be the most efficient way to tune models and reduce errors… we are working on it…
Current work and future plans

- Perform impact experiments with real observations on the NCEP system (this year)
- Install the LEKF on the NASA fvGCM (with Atlas and Todling)
- Perform adaptive observations and AIRS impact experiments (next year)
- Check impact of 4D LEKF (assimilation of asynchronous obs)
- Model deficiencies: estimate error using nudging, expand into state dependent order EOFs, use LEKF to estimate amplitudes
- We hope our system will become operational at NCEP!
- It should be cheap enough to be collocated with the space instruments for adaptive observations